DISCRETE CURVATURE ENERGIES

SEBASTIAN SCHOLTES

Abstract. We investigate the relationship between a curvature energy $\mathcal{F}$, for example the Möbius energy or thickness, and discrete versions $\mathcal{F}_n$ of this energy. These discrete energies $\mathcal{F}_n$ are defined on equilateral polygons with n vertices, while the energy $\mathcal{F}$ is initially defined on all curves, but finite energy usually assures that the curve has some higher regularity, i.e. the arc length parametrisation typically belongs to some (fractional) Sobolev space. It will turn out that the energy $\mathcal{F}$ is the Γ-limit of the corresponding discrete energies $\mathcal{F}_n$ for $n \to \infty$. This directly implies the convergence of almost minimizers of the discrete energies in a fixed knot class to a minimizer of the smooth energy in the same knot class, where, depending on which energy $\mathcal{F}$ represents in particular, we additionally might have to guarantee that the limit of the polygons belongs to the same knot class. Moreover, we show that the unique absolute minimizer of the discrete energies is the regular n-gon.

Institut f. Mathematik, RWTH Aachen University, Templergraben 55, D-52062 Aachen, Germany
E-mail address: sebastian.scholtes@rwth-aachen.de
URL: http://www.instmath.rwth-aachen.de/~scholtes/home/

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