Regularization by noise for scalar conservation laws

Mario Maurelli*
Joint work with Benjamin Gess

MPI Leipzig, November 30, 2017

Abstract

We say that a regularization by noise phenomenon occurs for a possibly ill-posed differential equation if this equation becomes well-posed (in a pathwise sense) under addition of noise. Most of the results in this direction are limited to SDEs and associated linear SPDEs.

In this talk, we show a regularization by noise result for a nonlinear SPDE, namely a stochastic scalar conservation law on $\mathbb{R}^d$ with a space-irregular flux:

$$\partial_t v + b \cdot \nabla[v^2] + \nabla v \circ \dot{W} = 0,$$

where $b = b(x)$ is a given deterministic, possibly irregular vector field, W is a d-dimensional Brownian motion ($\circ$ denotes Stratonovich integration) and $v = v(t, x, \omega)$ is the scalar solution. More precisely we prove that, under suitable Sobolev assumptions on b and integrability assumptions on its divergence, the equation admits a unique entropy solution. The result is false without noise.

The proof of uniqueness is based on a careful combination of arguments used in the linear case: first we show the renormalization property for the kinetic formulation of the equation, then we use second order PDE estimates and a duality argument to conclude.

If time permits, we will discuss also some open questions.

*Weierstrass Institute for Applied Analysis and Stochastics, Berlin, & Technische Universität Berlin