Absolute Continuity under Time Shift and Related Stochastic Calculus

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Abstract: The talk is concerned with a class of two-sided stochastic processes of the form $X = W + A$. Here W is a two-sided Brownian motion with random initial data at time zero and $A \equiv A(W)$ is a function of W. Elements of the related stochastic calculus are introduced. In particular, the role of finite dimensional projections arising from the Lévy-Ciesielski representation of the Brownian motion are discussed. The calculus is adjusted to the case when A is a jump process.

Absolute continuity of $(X, P_{\boldsymbol{\nu}})$ under time shift of trajectories is investigated. For example under various conditions on the initial density with respect to the Lebesgue measure, $m = d\boldsymbol{\nu}/dx$, and on A with $A_0 = 0$ we verify

$$\frac{P_{\boldsymbol{\nu}}(dX_{\cdot - t})}{P_{an\boldsymbol{\nu}}(dX_{\cdot})} = \frac{m(X_{-t})}{m(X_0)} \cdot \prod_i |\nabla_{W_0} X_{-t}|_i$$

a.e. where the product is taken over all coordinates. Here $\sum_i (\nabla_{W_0} X_{-t})_i$ is the divergence of X_{-t} with respect to the initial position. Crucial for this is the *temporal homogeneity* in the sense that $X(W_{\cdot + v} + A_v \mathbb{I}) = X_{\cdot + v}(W)$, $v \in \mathbb{R}$, where $A_v \mathbb{I}$ is the trajectory taking the constant value $A_v(W)$.

By means of a such a density, partial integration relative to the generator of the process X is established. A further application is relative compactness of sequences of processes of the form $X^n = W + A^n$, $n \in \mathbb{N}$.

References

[1] J.-U. Löbus (2017). Absolute Continuity under Time Shift of Trajectories and Stochastic Calculus, *Memoirs of the American Mathematical Society*, **249**, No. 1185 (2017).