Steiner representations of hypersurfaces

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Abstract. Let $X \subseteq \mathbb{P}^{n+1}$ be an integral hypersurface of degree d. The description of hypersurfaces in $\mathbb{P}^{n+1}$ as zero loci of suitable square matrices (possibly with some further properties, e.g. with linear entries, symmetric, skew–symmetric, etc.) is a very classical topic in algebraic geometry. In this talk we show that each locally Cohen–Macaulay instanton sheaf $\mathcal{E}$ on X with respect to $\mathcal{O}_X \otimes \mathcal{O}_{\mathbb{P}^{n+1}}(1)$ yields the existence of Steiner bundles $\mathcal{G}$ and $\mathcal{F}$ on $\mathbb{P}^{n+1}$ of the same rank r and a morphism $\varphi\colon \mathcal{G}(-1) \to \mathcal{F}^\vee$ such that the form defining X to the power $\mathrm{rk}(\mathcal{E})$ is exactly $\det(\varphi)$. In particular, we show that the form defining a smooth integral surface in $\mathbb{P}^3$ is the pfaffian of some skew–symmetric morphism $\varphi\colon \mathcal{F}(-1) \to \mathcal{F}^\vee$, where $\mathcal{F}$ is a suitable Steiner bundle on $\mathbb{P}^3$ of sufficiently large even rank. Finally we deal with the case of cubic fourfolds in $\mathbb{P}^5$, showing how the existence of Steiner pfaffian representations is related to the existence of particular subvarieties of the cubic. This is a joint work with Gianfranco Casnati.