Let's get $\mathbb{R}$eal
Exercise session part 4

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1. Compute the sequence of moments of
   (i) the cube $[-1,1]^3 \subset \mathbb{R}^3$;
   (ii) the disc $\{(x,y) \in \mathbb{R}^2 \,|\, x^2+y^2 \leq 1\}$.
2. Explore the new type of Stokes constraints in section 4 of the paper[1] by Matteo Tacchi, Jean Bernard Lasserre, and Didier Henrion on *Stokes, Gibbs, and Volume Computation of Semi-Algebraic Sets*.
3. What is `GloptiPoly 3`?
4. Let $S = \{(x,y,z) \in \mathbb{R}^3 \,|\, 1 - x^4 - y^4 - z^2 - 2xyz \geq 0\}$, displayed in Figure 1. Approximate the volume of S using the methods discussed in class. Write down the semidefinite program (with and without Stokes constraints) and solve it for low degrees.

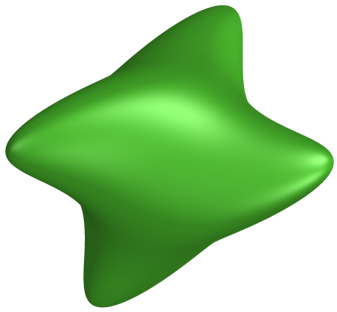

Figure 1: The set S from Exercise 4

[1] https://doi.org/10.1007/s00454-022-00462-0

5. What is the relation between primal and dual linear programs? What is the relation between primal and dual semidefinite programs? How does this translate into geometry? Explore the book[2] edited by Grigoriy Blekherman, Pablo A. Parrilo, and Rekha R. Thomas on *Semidefinite Optimization and Convex Algebraic Geometry*. See Chapter 5.

6. What are other methods to compute the volume of a semialgebraic set? In the paper[3] by Pierre Lairez, Marc Mezzarobba, and Mohab Safey El Din on *Computing the Volume of Compact Semi-Algebraic Sets*, the authors develop a method which is based on periods, Picard-Fuchs differential equation, and differential algebra more in general.

[2] https://doi.org/10.1137/1.9781611972290
[3] https://doi.org/10.1145/3326229.3326262