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Abstract: We discuss the precise relation of the open N=2 string to a self-dual Yang-Mills (SDYM) system in 2+2 dimensions. In particular, we review the description of the string target space action in terms of SDYM in a "picture hyperspace" parametrised by the standard vectorial $\mathbb{R}^{2,2}$ coordinate together with a commuting spinor of $SO(2,2)$. The component form contains an infinite tower of prepotentials coupled to the one representing the SDYM degree of freedom. The truncation to five fields yields a novel one-loop exact lagrangean field theory.

1. Introduction The relation to self-dual Yang-Mills (SDYM) of the critical open N=2 string has recently been elaborated by us [1] in view of the particular picture degeneracy and global $SO(2,2)$ properties of the physical spectrum of string states. There has been much discussion in the literature of this relationship since Ooguri and Vafa [2] first mooted the idea that the self-duality equations,

$$F_{\mu\nu} - \tfrac{1}{2}\epsilon_{\mu\nu}{}^{\rho\lambda}F_{\rho\lambda} \;=\; 0\;, \tag{1}$$

with the field strengths taking values in the Chan-Paton Lie algebra, describe what at that stage appeared to be the single dynamical degree of freedom of the open N=2 string. (We shall not give all relevant references here, referring to [1] for further references). The comparison has been based on determinations of tree-level amplitudes for the two theories, so light-cone gauge action principles for SDYM have played a central role in the discussion. In two-spinor notation, using the splitting of the $\mathbb{R}^{2,2}$ "Lorentz algebra",

$$so(2,2)\;\cong\; sl(2,\mathbb{R})\oplus sl(2,\mathbb{R})' \quad\Longleftrightarrow\quad x^\mu\sigma_\mu^{\alpha\dot\beta} \;=\; x^{\alpha\dot\beta} \;=\; \begin{pmatrix} x^0+x^3 & x^1+x^2 \\ x^1-x^2 & x^0-x^3 \end{pmatrix}, \tag{2}$$

the three (real) SDYM equations take the form

$$F_{\alpha\beta} \;\equiv\; \tfrac{1}{2}\left(\partial_{(\alpha}{}^{\dot\gamma}A_{\beta)\dot\gamma} \;+\; [A_\alpha{}^{\dot\gamma}, A_{\beta\dot\gamma}]\right) \;=\; 0 \tag{3}$$

In components, with the spinor indices α, β taking values $+$ and $-$, we have

$$
\begin{aligned}
F^{++} &\equiv \partial^{+\dot\gamma} A^{+}_{\dot\gamma} + \tfrac{1}{2}[A^{+\dot\gamma}, A^{+}_{\dot\gamma}] = 0 \\
F^{+-} &\equiv \tfrac{1}{2}\Big(\partial^{+\dot\gamma} A^{-}_{\dot\gamma} + \partial^{-\dot\gamma} A^{+}_{\dot\gamma} + [A^{+\dot\gamma}, A^{-}_{\dot\gamma}]\Big) = 0 \\
F^{--} &\equiv \partial^{-\dot\gamma} A^{-}_{\dot\gamma} + \tfrac{1}{2}[A^{-\dot\gamma}, A^{-}_{\dot\gamma}] = 0 \quad .
\end{aligned} \tag{4}
$$

Clearly, the $(++)$ equation affords the generalised light-cone gauge $A^{+}_{\dot\gamma}=0$ in which F^{+-} becomes homogeneous. Two strategies now suggest themselves. First, resolving the (inhomogeneous) $(--)$ equation in the Yang fashion,

$$A^{-}_{\dot\alpha} \;=\; e^{-\phi}\, \partial^{-}_{\dot\alpha}\, e^{+\phi} \;, \tag{5}$$

the $(+-)$ equation describes the ϕ-dynamics in the form of the (non-polynomial) Yang equation

$$\partial^{+\dot\alpha}\, (e^{-\phi}\, \partial^{-}_{\dot\alpha}\, e^{+\phi}) \;=\; 0 \;. \tag{6}$$

Second, the (homogeneous) $(+-)$ equation is instead fulfilled in terms of a Leznov prepotential, writing

$$A^{-}_{\dot\alpha} \;=\; \partial^{+}_{\dot\alpha}\, \varphi^{--} \;, \tag{7}$$

which then must satisfy $F^{--}=0$, tantamount to the (quadratic) Leznov equation,

$$\Box\varphi^{--} - \tfrac{1}{2}[\partial^{+\dot\alpha}\varphi^{--}, \partial^{+}_{\dot\alpha}\varphi^{--}] \;=\; 0 \;. \tag{8}$$

The light-cone gauge explicitly breaks the global $SO(2,2)$ covariance of eq. (3) to $GL(1,\mathbb{R}) \otimes SL(2,\mathbb{R})'$. In a Cartan-Weyl basis for $sl(2,\mathbb{R})$ consisting of a diagonal hyperbolic generator L_{+-} and two parabolic generators $L_{\pm\pm}$, the unbroken $gl(1,\mathbb{R})$ generator is L_{+-} in the Yang but L_{++} in the Leznov case.

Non-covariant action principles for (6) or (8) yield themselves using merely the prepotentials,

$$S_{\text{Yang}} \;=\; \mu^2 \int d^4x \;\mathrm{Tr}\Big\{-\tfrac{1}{2}\;\phi\Box\phi \;+\; \tfrac{1}{3}\;\phi\;\partial^{(+\dot\alpha}\phi\;\partial^{-)}_{\dot\alpha}\phi \;+\; \mathcal{O}(\phi^4)\Big\} \tag{9}$$

$$S_{\text{Leznov}} \;=\; \mu^2 \int d^4x \;\mathrm{Tr}\Big\{-\tfrac{1}{2}\;\varphi^{--}\Box\varphi^{--} \;+\; \tfrac{1}{6}\;\varphi^{--}\Big[\partial^{+\dot\alpha}\varphi^{--}, \partial^{+}_{\dot\alpha}\varphi^{--}\Big]\Big\} \tag{10}$$

with some mass scale μ. Alternatively, Lagrange multipliers facilitate the construction of dimensionless actions, for example,

$$S_{\text{CS}} \;=\; \int d^4x \;\mathrm{Tr}\Big\{-\varphi^{++}\Box\varphi^{--} \;+\; \varphi^{++}\Big[\partial^{+\dot\alpha}\varphi^{--}, \partial^{+}_{\dot\alpha}\varphi^{--}\Big]\Big\} \tag{11}$$

which was shown to be even one-loop exact by Chalmers and Siegel [3].

The tree-level amplitudes following from these actions are extremely simple. Since we are dealing with massless fields in 2+2 dimensions, the on-shell momenta factorise,

$$k^{\alpha\dot\beta}\; k_{\alpha\dot\beta} \;=\; 0 \qquad\Longleftrightarrow\qquad k^{\alpha\dot\beta} \;=\; \kappa^{\alpha}\;\kappa^{\dot\alpha} \quad . \tag{12}$$

The on-shell three-point functions $A_3(k_1,k_2,k_3)$ can be read off as

$$A_3^{\rm Yang} \;=\; f^{abc}\,\kappa_1^{(+}\kappa_2^{-)}\;\kappa_1^{\dot\alpha}\kappa_{2\dot\alpha} \qquad , \qquad A_3^{\rm Leznov} \;=\; f^{abc}\,\kappa_1^{+}\kappa_2^{+}\;\kappa_1^{\dot\alpha}\kappa_{2\dot\alpha} \tag{13}$$

where $\sum_i k_i = 0$ and f^{abc} are the structure constants of the gauge group. Surprisingly, the four-point Feynman diagrams sum to zero, in virtue of a quartic contact interaction in the Yang case. It is believed that all higher tree amplitudes vanish on-shell. The version of Chalmers and Siegel leads to the same tree-level amplitudes as the Leznov action, although in the former case one of the legs needs to be the multiplier field. Interestingly, this two-field theory does not allow diagrams beyond one loop. As we shall describe below, both (10) and (11) are related to the target space effective action for the open N=2 string.

2. N=2 Open Strings The spectrum of world-sheet fields in the NSR formulation of N=2 strings consists of the 2d N=2 supergravity multiplet, whose conformal gauge fixing produces the standard set of N=2 superconformal ghosts, plus N=2 matter fields $(X^{\alpha\dot\beta},\Psi^{\alpha\dot\beta})$. The computation of open string amplitudes requires the evaluation of correlation functions for appropriate choices of physical external states on Riemann surfaces with handles, boundaries, punctures, and a harmonic $U(1)$ gauge field background with instantons. The result is to be integrated over the moduli of the Riemann surface and the $U(1)$ gauge field, and finally one is to sum over the topologies labelled by the Euler and $U(1)$ instanton numbers. The relative cohomology of the BRST operator determines the string external states, which are annihilated by the commuting N=2 Virasoro and the anticommuting N=2 antighost zero modes. The resulting spectrum has the following quantum numbers:

- total ghost number $u\in\mathbb{Z}$
- target space momentum $k^{\alpha\dot\beta}$
- total picture $\pi\in\mathbb{Z}$
- picture twist $\Delta\in\mathbb{R}$
- $gl(1,\mathbb{R})\oplus sl(2,\mathbb{R})'$ quantum numbers $m,\ (j',m')$

These quantum numbers are however redundant, since they have interrelationships: $k{\cdot}k=0$ (i.e. $k^{\alpha\dot\beta}{=}\kappa^\alpha\kappa^{\dot\beta}$), $u{-}\pi=1$, $j'=m'=0$, and m runs in integral steps from $-j$ to $+j$, where $j:=\frac{\pi}{2}{+}1$. The physical spectrum consists of just one $SL(2,\mathbb{R})'$ singlet for each value of π, Δ, and $(\kappa,\dot\kappa)$. There is still a certain redundancy, since the pictures $(\pi\geq\pi_0,\Delta)$ can be reached from $(\pi_0,0)$ by applying spectral flow $\mathcal{S}$ and picture raising $\mathcal{P}^\alpha$, which commute with the BRST operator and effect the mappings

$$(\pi,\Delta)\;\stackrel{\mathcal{S}(\rho)}{\longrightarrow}\;(\pi,\Delta{+}2\rho) \qquad , \qquad (\pi,\Delta)\;\stackrel{\mathcal{P}^\alpha}{\longrightarrow}\;(\pi{+}1,\Delta) \tag{14}$$

with $\rho\in\mathbb{R}$. Because the string path integral integrates over the twists of the $U(1)$ gauge bundle, it averages over the spectral flow orbits. The $\mathcal{S}$-equivalent states therefore ought to be identified and we may choose the Δ=0 representative. Picture lowering can also be constructed, except on zero-momentum states. In essence, all physical states (with $k{\neq}0$) can be generated starting from the canonical picture $\pi{=}-2$ (i.e. $j{=}0$), and the result is symmetric under the "Poincaré duality" $\pi \stackrel{*}{\to} -4-2\pi$ (i.e. $j \stackrel{*}{\to} -j$):

$$\begin{array}{c|cccccccccc} \pi & \cdots & -5 & -4 & -3 & -2 & -1 & 0 & +1 & \cdots \\ j & \cdots & -\frac{3}{2} & -1 & -\frac{1}{2} & 0 & +\frac{1}{2} & +1 & +\frac{3}{2} & \cdots \\ \hline \text{states} & \cdots & |\alpha\beta\gamma\rangle^* & |\alpha\beta\rangle^* & |\alpha\rangle^* & |\,\rangle & |\alpha\rangle & |\alpha\beta\rangle & |\alpha\beta\gamma\rangle & \cdots \end{array} \quad . \tag{15}$$

The states form $SL(2,\mathbb{R})$ tensors of rank $2|j|$ (spin $|j|$), because the picture-raising operator $\mathcal{P}^\alpha$ carries a spinor index. There is no contradiction with the above statement of unit multiplicity, since all states in a given $SL(2,\mathbb{R})$ multiplet are related to each other, albeit in a non-local fashion. The open spinor indices are just carried by normalisation factors multilinear in κ^α, with $\mathcal{P}^\alpha$ increasing the spin by $\frac{1}{2}$:

$$\mathcal{P}^{\alpha_1}\mathcal{P}^{\alpha_2}\ldots\mathcal{P}^{\alpha_{2j}}\;|(0);k\rangle \;=\; |\alpha_1\alpha_2\ldots\alpha_{2j};k\rangle \;\propto\; \kappa^{\alpha_1}\kappa^{\alpha_2}\ldots\kappa^{\alpha_{2j}}\;\;|(j);k\rangle \quad . \tag{16}$$

The NSR formulation of $N{=}2$ strings introduces a complex structure in the target space, which explicitly breaks $SO(2,2)\to GL(1,\mathbb{R})\otimes SL(2,\mathbb{R})'$. Individual pieces of an n-point amplitude are only $SL(2,\mathbb{R})'$ invariant, and contributions from the M-instanton $U(1)$ background carry a $gl(1,\mathbb{R})$ weight equal to M. Surprisingly, the path integral measure constrains the instanton sum to $|M|\leq J\equiv n{-}2$ at tree level. Moreover, the weight factors built from the string coupling e and the instanton angle θ conspire to restore $SO(2,2)$ invariance of the instanton sum if $\sqrt{e}(\cos\frac{\theta}{2},\sin\frac{\theta}{2})$ is assumed to transform as an $SL(2,\mathbb{R})$ spinor! This spinor simply parametrises the choices of complex structure, and it may be Lorentz-rotated to $(1,0)$. Henceforth we shall remain in such a frame where $e{=}1$ and $\theta{=}0$. It has the virtue that only the highest $SL(2,\mathbb{R})$ weights $m_i{=}j_i$, $i=1,\ldots,n$, occur and only the maximal instanton number sector, $M=J$, contributes.

The tree-level open string on-shell amplitudes may then be found to be

$$\begin{aligned} A_3^{\text{string}} &= f^{abc}\;\kappa_1^+\kappa_2^+\;\kappa_1^{\dot\alpha}\kappa_{2\dot\alpha} \;=\; A_3^{\text{Leznov}} && (17)\\ A_4^{\text{string}} &\propto \kappa_1^+\kappa_2^+\kappa_3^+\kappa_4^+(\dot\kappa_1{\cdot}\dot\kappa_2\;\dot\kappa_3{\cdot}\dot\kappa_4\;t+\dot\kappa_2{\cdot}\dot\kappa_3\;\dot\kappa_4{\cdot}\dot\kappa_1\;s) \;=\; 0 && (18)\\ A_{n>4}^{\text{string}} &= 0 \quad . && (19) \end{aligned}$$

They are independent of the external $SL(2,\mathbb{R})$ spins j_i, as long as $\sum_{i=1}^n j_i = J\equiv n{-}2$. Clearly, the Leznov version (13) of SDYM is reproduced. However, a covariant description needs to take the entire tower of states in (15) into account.

3. Target Space Actions Physical string states correspond to target space (background) fields, whose on-shell dynamics is determined by the string scattering amplitudes. In particular, the string three-point functions directly yield cubic terms in the effective target space action. In the present case, the correspondence reads

$$|{+}{+}\ldots{+}\rangle \;\longleftrightarrow\; \varphi^{--\cdots-} \quad (j{\geq}0) \quad , \qquad |{-}{-}\ldots{-}\rangle^* \;\longleftrightarrow\; \varphi^{++\cdots+} \quad (j{<}0) \tag{20}$$

and we denote the fields by φ_j. Then, the target space effective action for the infinite tower $\{\varphi_j\}$ is

$$
\begin{aligned}
S_\infty &= \int d^4x \ \mathrm{Tr}\Big\{ -\tfrac{1}{2} \sum_{j\in\mathbb{Z}/2} \varphi_{(-j)} \Box \varphi_{(+j)} \ + \ \tfrac{1}{3} \sum_{j_1+j_2+j_3=1} \varphi_{(j_1)} \Big[\partial^{+\dot\alpha} \varphi_{(j_2)} \ , \ \partial^{+}_{\dot\alpha} \varphi_{(j_3)} \Big] \Big\} \\
&= \int d^4x \ \mathrm{Tr}\Big\{ -\tfrac{1}{2} \ \Phi^{--} \Box \Phi^{--} \ + \ \tfrac{1}{6} \ \Phi^{--} \Big[\partial^{+\dot\alpha} \Phi^{--} , \partial^{+}_{\dot\alpha} \Phi^{--} \Big] \Big\}_{\eta^4}
\end{aligned} \tag{21}
$$

where we have introduced a "picture hyperfield",

$$
\Phi^{--}(x,\eta^-) \ = \ \sum_j \ (\eta^-)^{2j} \ \varphi_{1-j}(x) \quad , \tag{22}
$$

depending on an extra commuting coordinate η^-, and we project the Lagrangean onto the part quartic in η. It is remarkable that the action (21) has the Leznov form in terms of the hyperfield. It not only reproduces all (tree-level) string three-point functions (17) but also yields vanishing four- and probably higher-point functions for the same reason that the Leznov action (10) does. Picture raising induces a dual action on the component fields,

$$
Q^+ : \quad \varphi_j \ \longrightarrow \ (3{-}2j) \ \varphi_{j-\frac{1}{2}} \quad , \tag{23}
$$

which is nothing but the η^- derivative on the hyperfield!

Three successive truncations to a finite number of fields are possible. First, keeping only $\{\varphi_{-1}, \varphi_{-\frac{1}{2}}, \varphi_0, \varphi_{+\frac{1}{2}}, \varphi_{+1}\}$, a consistent five-field model ensues, viz.,

$$
\begin{aligned}
S_5 \ = \ \int d^4x \ \mathrm{Tr}\Big\{ \ & \tfrac{1}{2} \partial^{+\dot\alpha} \varphi \ \partial^{-}_{\dot\alpha} \varphi \ + \ \partial^{+\dot\alpha} \varphi^{+} \partial^{-}_{\dot\alpha} \varphi^{-} \ + \ \partial^{+\dot\alpha} \varphi^{++} \partial^{-}_{\dot\alpha} \varphi^{--} \\
& + \tfrac{1}{2} \varphi \ [\partial^{+\dot\alpha} \varphi^{-} , \partial^{+}_{\dot\alpha} \varphi^{-}] \ + \ \tfrac{1}{2} \varphi^{--} [\partial^{+\dot\alpha} \varphi \ , \partial^{+}_{\dot\alpha} \varphi] \\
& + \varphi^{--} [\partial^{+\dot\alpha} \varphi^{+} , \partial^{+}_{\dot\alpha} \varphi^{-}] \ + \ \tfrac{1}{2} \varphi^{++} [\partial^{+\dot\alpha} \varphi^{--} , \partial^{+}_{\dot\alpha} \varphi^{--}] \ \Big\} \quad .
\end{aligned} \tag{24}
$$

Second, eliminating also the fermions leaves us with three fields. Third, we may in addition kill φ_0 as well, resulting in the two-field model of Chalmers and Siegel [3]! All truncations share the one-loop exactness mentioned before.

4. Self-Duality in Hyperspace The infinite tower of higher-spin fields which arise from the picture degeneracy parametrise simply SDYM in a hyperspace with coordinates $\{x^{\alpha\dot\alpha}, \eta^\alpha, \bar\eta^{\dot\alpha}\}$, with η and $\bar\eta$ *commuting* spinors. This commutative variant of superspace exhibits a $\mathbb{Z}_2$-graded Lie-algebra variant of the super-Poincare algebra (i.e. with all anti-commutators replaced by commutators). So the covariant target space symmetry is effectively the extension of the $\mathbb{R}^{2,2}$ Poincaré algebra by two *Grassmann-even* spinorial generators squaring to a translation, i.e., $[Q_{\dot\alpha}, Q_\alpha] = P_{\alpha\dot\alpha}$ (see [1] for details). Hyperspace self-duality allows compact expression in a chiral subspace independent of the $\bar\eta$ coordinates. In terms of chiral subspace gauge-covariant derivatives $\mathcal{D}_\alpha = \partial_\alpha + A_\alpha(x,\eta)$ and $\mathcal{D}_{\alpha\dot\alpha} = \partial_{\alpha\dot\alpha} + A_{\alpha\dot\alpha}(x,\eta)$, the self-duality conditions take the simple form

$$[\mathcal{D}_\alpha, \mathcal{D}_\beta] = \epsilon_{\alpha\beta} F \quad , \quad [\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}] = \epsilon_{\alpha\beta} F_{\dot\beta} \quad , \quad [\mathcal{D}_{\alpha\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] = \epsilon_{\alpha\beta} F_{\dot\alpha\dot\beta} \quad . \tag{25}$$

Jacobi identities yield the equations

$$\mathcal{D}_\alpha{}^{\dot\alpha} F_{\dot\alpha\dot\beta} = 0 \qquad , \qquad \mathcal{D}_\alpha{}^{\dot\alpha} F_{\dot\alpha} = 0 \qquad , \qquad \mathcal{D}_{\alpha\dot\alpha} F = \mathcal{D}_\alpha F_{\dot\alpha} \quad . \tag{26}$$

The first two are respectively the Yang-Mills and Dirac equations for a SDYM multiplet, and the third implies the scalar field equation $\mathcal{D}^2 F = [F^{\dot\alpha}, F_{\dot\alpha}]$. All chiral hyperfields have η expansions, e.g.

$$A_\alpha(x,\eta) \;=\; A_\alpha(x) \;+\; \eta^\beta A_{\alpha\beta}(x) \;+\; \eta^\beta\eta^\gamma A_{\alpha\beta\gamma}(x) \;+\; \ldots \quad . \tag{27}$$

Choosing the light-cone gauge, $A^+ = 0 = A^+_{\dot\alpha}$, we note that all fields are defined in terms of a generalised Leznov prepotential,

$$A^- \;=\; \partial^+\,\Phi^{--} \qquad , \qquad A^-_{\dot\alpha} \;=\; \partial^+_{\dot\alpha}\,\Phi^{--} \quad , \tag{28}$$

$$F \;=\; \partial^+\partial^+\,\Phi^{--} \quad , \quad F_{\dot\alpha} \;=\; \partial^+\partial^+_{\dot\alpha}\,\Phi^{--} \quad , \quad F_{\dot\alpha\dot\beta} \;=\; \partial^+_{\dot\alpha}\partial^+_{\dot\beta}\,\Phi^{--} \quad . \tag{29}$$

Since ∂^- does not occur in the above, all fields are determined by the chiral (η^+–independent) part of Φ^{--}. The dynamics is determined by the remaining constraints

$$[\mathcal{D}^-_{\dot\alpha}, \mathcal{D}^-_{\dot\beta}] \;=\; 0 \qquad \text{and} \qquad [\mathcal{D}^-, \mathcal{D}^-_{\dot\beta}] \;=\; 0 \;, \tag{30}$$

where the former equation is precisely the Leznov equation for Φ^{--}. Choosing this to be chiral, $\Phi^{--} = \Phi^{--}(x,\eta^-)$, allows identification with (22), with action given by (21). The second equation above then merely determines the η^- dependence of Φ^{--}.

The restricted system of five fields (24) has the $SO(2,2)$-invariant action

$$S_5^{\rm inv} \;=\; \int d^4x \; \mathrm{Tr}\Big\{ \tfrac14 g^{\alpha\beta} F_{\alpha\beta} \;+\; \tfrac13 \chi^\alpha \mathcal{D}_{\alpha\dot\alpha} F^{\dot\alpha} \;+\; \tfrac18 \mathcal{D}^{\alpha\dot\alpha} F\, \mathcal{D}_{\alpha\dot\alpha} F \;+\; \tfrac12 F\,[F^{\dot\alpha}, F_{\dot\alpha}] \Big\} \tag{31}$$

where $g^{\alpha\beta}$ and χ^α are (propagating) multiplier fields for $A_{\alpha\dot\alpha}$ and $F_{\dot\alpha}$, respectively. The similarity with N=4 supersymmetric SDYM [4] is evident, however with commuting single-multiplicity fermions replacing multiplicity 4 anticommuting ones.

To conclude, we note that theories of N=2 closed as well as N=(2, 1) heterotic strings are also intimately related to self-dual geometry and our covariant hyperspace description generalises to both these cases.

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